HYDRODYNAMIC THERMAL BURST IN RADIAL BEARING

Yu. A. Buevich and M. I. Zaslavskii

UDC 621.891

The work involves the investigation of the conditions of occurrence of a thermal burst in an unloaded bearing with a pseudoplastic liquid. It is shown that in a loaded bearing a local thermal burst is possible.

The possibility of unbounded growth of dissipative heat release in the flow of a liquid with strong nonlinear dependence of the viscosity on the temperature, hindering the establishment of stready-state flow conditions, was apparently pointed out for the first time in [1-3]. Since then the realization of thermal bursts taking into account the dissipative heating and heat exchange with the environment has been investigated for many types of flow of liquids with different rheological properties (see, e.g., [4-8]).

Among the flows, in which the effects of internal heat release are particularly important, belong flows of thin layers of lubricant in various types of bearing [9]. These effects greatly change the parameters of such flows. In particular, increase of the maximum temperature in the lubricant layer as a result of thermal burst above some limiting permissible value and also the decrease of the minimum thickness of the layer up to contact of the lubricated solid surfaces as a result of disastrous drop of viscosity caused by the heating may lead to early breakdown of bearings.

Here we will examine the plane problem for a radial bearing of unbounded length. The steady-state equations of the conservation of energy and impulse in the approximation of a thin layer have the form

$$\rho c\frac{\partial}{\partial x}(uT)=\lambda\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2,\quad \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)=\frac{dp}{dx}. \tag{1}$$

Using the condition of adhesion and assuming that the thermal conductivity of the materials of the shaft and of the bearing housing greatly exceeds λ, we will represent the boundary conditions on the surface of the bearing (y = 0) and of the shaft (y = h) in the form

$$T=T_0,\ u=0\ (y=0);\quad T=T_1,\ u=U=\Omega R\,(y=h). \tag{2}$$

In addition to that, below we will examine the problem in which the first condition in (2) is replaced by the equality to zero of the heat flow to the bearing (i.e., $\partial T/\partial y = 0$ for y = h). The system with heat-insulated bearing simulates real systems fairly well [9].

Additionally to (2) we impose the ordinary condition of periodicity on all functions from the longitudinal coordinate $x = R\varphi$, replacing in our case the "initial" conditions [10].

The dependence of viscosity on the temperature and slip velocity will be described by the formula

$$\mu=K\left(\frac{1}{\gamma}\left|\frac{\partial u}{\partial y}\right|\right)^{n-1}\exp\left[-m(T-T_1)\right], \tag{3}$$

which is approximately correct for graded pseudoplastic (n < 1) and dilatable (n > 1) liquids; γ is the characteristic slip velocity. To a Newtonian liquid corresponds n = 1, $K = \mu_1$. For the sake of determinacy, the values of K and μ_1 are reduced to the shaft temperature T_1.

If the temperature gradients longitudinally and transversely are of the same order of magnitude, we have

$$\rho c\frac{\partial}{\partial x}(uT)/\lambda\frac{\partial^2 T}{\partial y^2}\sim\frac{\rho c U h^2}{\lambda R}=\mathrm{Pe}\frac{h}{R},\quad \mathrm{Pe}=\frac{Uh}{a}, \tag{4}$$

so that when the layer of lubricant is sufficiently thin, the term in the left-hand part of

---

A. M. Gorky Urals State University, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 42, No. 5, pp. 813-819, May, 1982. Original article submitted May 12, 1981.

 0022-0841/82/4205-0566$07.50

the first equation (1) may be neglected; in this case, only the dependence of u and T on y is important. The opposite boundary case, when the number Bi = $\alpha h/\lambda \ll 1$ (here, α is the effective heat transfer coefficient), so that the temperature may be considered to be approximately uniform, was examined [6, 7] in connection with the problem of establishing steady-state Couette flows. The dependence of the temperature on the transverse coordinate may apparently be neglected under artificially impaired heat removal across the flow boundary, as occurred in the experiments [7]. However, it is not adequate to the situation in a bearing where, even if we neglect the heat removal by the lubricant itself, the heat is rapidly absorbed by the massive wall with high heat capacity and possibly also by the bearing housing, where in the general case $\Delta T = T_1 - T_0 \neq 0$ applies.

With the thickness h of the lubricant layer depending on x, when $dp/dx \neq 0$ and the condition of periodicity must be imposed on p, the full solution of the problem (1)-(3) for a bearing encounters considerable difficulty even when the estimate (4) is used. Therefore, authors dealing with the hydrodynamic theory of lubrication either neglect the inhomogeneity of the temperature field altogether ("isothermal" approximation), or they adopt various simplifying, albeit not always justified, assumptions: they examine the "adiabatic" approximation corresponding to neglecting the heat exchange with the walls, they neglect the effect of the pressure gradient on viscous energy dissipation, which corresponds to the assumption that the eccentricity of the bearing is relatively small, etc. (see, e.g., [9-11]).

Here we will examine first an "unloaded" bearing with a lubricant layer of constant thickness, filled with a liquid with μ from (3). In this case $dp/dx = 0$, so that from the second equation (1) follows:

$$\mu \frac{du}{dy} = C, \quad C = \frac{M}{2\pi R}, \tag{5}$$

$$\frac{du}{dy} = C^{1/n}\left(\frac{\gamma^{n-1}}{K}\right)^{1/n} \exp\frac{m(T-T_1)}{n}. \tag{6}$$

Here, the term in the left-hand part of the first equation (1) for such a bearing vanishes identically.

Using (5), (6) and introducing the dimensionless magnitudes

$$\theta = \frac{m}{n}(T-T_1), \quad \eta = \frac{y}{h}, \quad \beta^2 = \frac{mC^{1+1/n}h^2}{2n\lambda}\left(\frac{\gamma^{n-1}}{K}\right)^{1/n}, \tag{7}$$

we obtain from (1) and (2) the problem for the dimensionless temperature

$$\frac{d^2\theta}{d\eta^2} + 2\beta^2 e^{\theta} = 0; \quad \theta = \theta_0 = \frac{m}{n}\Delta T\,(\eta = 0); \ \theta = 0 \ (\eta = 1), \tag{8}$$

whose equation coincides with the principal equation of the steady-state theory of thermal burst in a flat vessel when there is a chemical reaction [12]. The solution of the problem (8) has the form

$$\varphi = e^{\theta} = \varphi_0\left[\frac{\operatorname{ch}\sigma}{\operatorname{ch}(\beta\sqrt{\varphi_0}\,\eta\operatorname{ch}\sigma - \sigma)}\right]^2, \quad \varphi_0 = \exp\theta_0, \tag{9}$$

and $\sigma(\beta, \varphi_0)$ is determined from the equation

$$\sqrt{\varphi_0}\operatorname{ch}\sigma = \operatorname{ch}(\beta\sqrt{\varphi_0}\operatorname{ch}\sigma - \sigma). \tag{10}$$

The relationships (9) and (10) are greatly simplified when the temperatures of the shaft and of the bearing housing are equal ($\varphi_0 = 1$), where

$$\varphi = \left[\frac{2\sigma}{\beta\operatorname{ch}(2\sigma\eta - \sigma)}\right]^2, \quad \frac{\sigma}{\operatorname{ch}\sigma} = \frac{\beta}{2}. \tag{11}$$

For a bearing with heat-insulated housing we obtain instead of (9), (10), or (11):

$$\varphi = \left[\frac{\sigma'}{\beta\operatorname{ch}(\sigma'\eta)}\right]^2, \quad \frac{\sigma'}{\operatorname{ch}\sigma'} = \beta. \tag{12}$$

The equations for σ and σ' have two real roots corresponding to steady-state regimes of flow only when β is smaller than some critical $\beta = \beta_*$. For the solutions of (11) and (12), β_* is equal to 1.330 and 0.665, respectively. The value of β_* for Eq. (10) is a monotonically decreasing function of φ_0 illustrated in Fig. 1. When $\beta > \beta_*$, a steady-state regime is

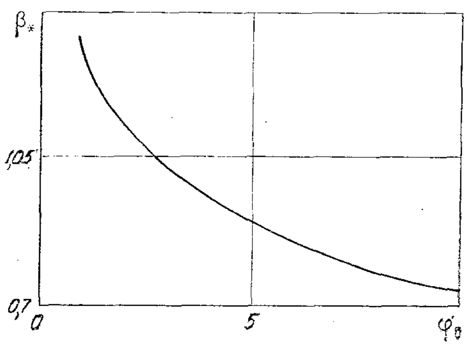

Fig. 1.

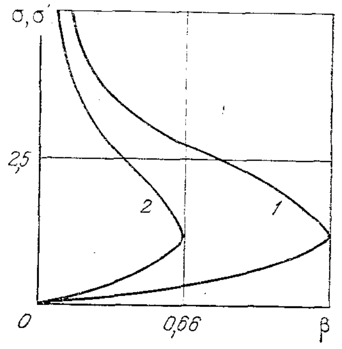

Fig. 2.

Fig. 1. Dependence of the critical parameter β_* on the dimensionless boundary temperature $\varphi_0 = \exp\theta_0$.

Fig. 2. Dependence of the roots σ and σ' (lower and upper branches of the curves, respectively) on β: 1) (11); 2) (12).

impossible: the heat transfer from the lubricant layer to the shaft and bearing is insufficient for compensating the dissipative heating of the liquid, and there is a progressive increase of the temperature of the liquid and the corresponding decrease of its viscosity. When $\beta < \beta_*$, there are two steady-state regimes, but stable is only the regime corresponding to the smaller root $\sigma(\beta)$ or $\sigma'(\beta)$; the dependence of these roots on β is shown in Fig. 2.

From the above relationships we can easily obtain the temperature and velocity fields for different values of the parameters:

$$T = T_1 + \frac{n}{m}\ln\varphi, \quad u = U\int\limits_0^\eta \varphi d\eta / \int\limits_0^1 \varphi d\eta, \tag{13}$$

and also an alternative representation for the constant

$$C = \frac{K}{\gamma^{n-1}}\left(\frac{U}{h}\right)^n \left(\int\limits_0^1 \varphi d\eta\right)^{-n}. \tag{14}$$

The characteristic profiles θ and u/U corresponding to the solutions of (11) and (12) are presented in Fig. 3.

It can be shown that with small β, the dependences (13) approach asymptotically those obtained from the linearized ($e^\theta \approx 1 + \theta$) problem (8).

We introduce the viscosity of the liquid at the surface of the shaft

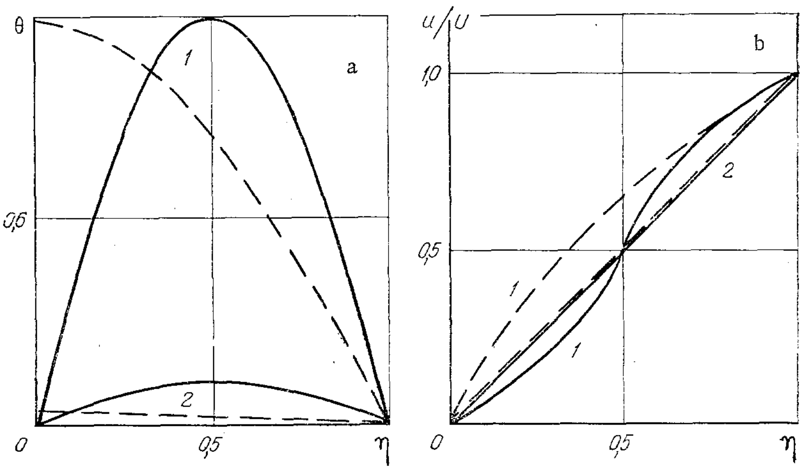

Fig. 3. Characteristic profiles θ (a) and u/U (b) for (11) and (12) (solid and hatched curves, respectively): 1) $\beta = \beta_*$; 2) $\beta = \beta_*/2$.

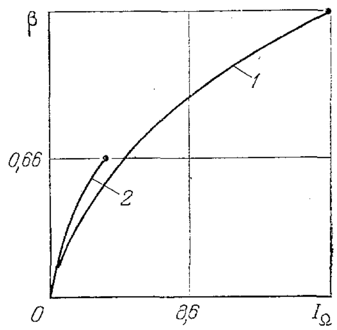

Fig. 4. The dependence of β on the regime parameter I_Ω: 1 and 2 correspond to the solutions of (11) and (12) respectively; the curves end at $\beta = \beta_*$.

$$\mu_1 = K\left(\frac{1}{\gamma}\left|\frac{du}{dy}\right|\right)^{n-1}_{\eta=1} \tag{15}$$

and examine the regimes corresponding to the specified torque M or the angular velocity of the shaft Ω. Calculating du/dy for η = 1 from (6) and using the formula for C in (5), the determination of β^2 in (7), and expression (15), we obtain the following condition of occurrence of a thermal burst in the regime M = const:

$$I_M = \frac{mM^2h^2}{n\lambda\mu_1} > 8\pi^2\beta_*^2. \tag{16}$$

When this condition is fulfilled, the angular velocity increases indefinitely without attaining a steady-state value. For situations corresponding to the solutions of (11) and (12), the right-hand part of (16) is equal to 140 and 35, respectively.

In the regime Ω = const, formula (14) must be used for C. The condition of occurrence of a thermal burst has the form

$$I_\Omega = \frac{m\mu_1\Omega^2R^2}{n\lambda} > 2\beta_*^2\left(\int_0^1 \varphi(\eta;\ \beta_*)\,d\eta\right)^2. \tag{17}$$

When this condition is fulfilled, the moment inhibiting rotation of the shaft approaches zero unboundedly. We point out that in distinction to the conclusion of [6], a hydrodynamic thermal burst is possible even in this regime. For the solutions of (11) and (12) the right-hand part of (17) is equal to 17.2 and 4.3, respectively.

If I_M or I_Ω are sufficiently small for a stable steady-state regime to exist, the values of β determining the temperature and velocity profiles are found from the equations

$$\beta = \sqrt{I_M}/2\sqrt{2}\,\pi, \quad \beta = f(I_\Omega), \tag{18}$$

and the functions $f(I_\Omega)$ corresponding to (11) and (12) are shown in Fig. 4.

Since β is an increasing function of I_M or I_Ω, other conditions being equal, β decreases with increasing exponent n in the rheological relationship (3). That means, in particular, that in the lubricant layer of the pseudoplastic liquid the thermal burst occurs more quickly with increasing M or Ω than in a layer of Newtonian liquid, and even more so in the case of a dilatable liquid. This last has to be taken into account when new types of lubricant are being worked out. In fact, it is usually desirable to weaken maximally the temperature dependence of the viscosity of lubricating liquids; this is often attained by adding high-molecular compounds to them [13, 14]. However, such additives endow the liquid with pseudoplastic properties, i.e., decrease of the parameter m occurs simultaneously with the decrease of n. Therefore, it is imperative to watch that the positive effect attained by weakening the dependence of viscosity on the temperature is not eliminated by the negative effect of strengthening its dependence on the slip velocity.

We want to point out that all the results obtained above are also fully applicable to the analysis of the effect of inner dissipative heating on the flows in rotational viscosimeters.

Let us now examine the flow in a loaded bearing with nonzero eccentricity, when $h = h(x)$, $dp/dx \neq 0$, assuming for the sake of simplicity that the lubricating liquid is Newtonian. In this case we have instead of (5)

$$\mu \frac{\partial u}{\partial y} = C + \frac{dp}{dx} y \tag{19}$$

and instead of Eq. (8)

$$\frac{\partial^2 \theta}{\partial \eta^2} + S(\eta) e^{\theta} = 0, \quad S(\eta) = 2\beta^2 (1 + \varkappa\eta)^2, \quad \varkappa = \frac{h}{C} \frac{dp}{dx}, \tag{20}$$

where θ, η, and β^2 and also the boundary conditions for (20) are determined, as before, by Eqs. (7) and (8) with $n = 1$. By substituting $\xi = (2\varkappa)^{-1}(1 + \varkappa\eta)^2$, Eq. (20) is reduced to the standard form of the equations originating in the steady-state theory of thermal burst (ignition) in systems with chemical reactions [12]:

$$\frac{\partial^2 \theta}{\partial \xi^2} + \frac{k}{\xi} \frac{\partial \theta}{\partial \xi} + 2\beta^2 e^{\theta} = 0, \tag{21}$$

where $k = 1/2$. This equation permits a solution in the known functions for $k = 0$ and $k = 1$; for other values of k a numerical solution is apparently indispensable. We want to point out that if, instead of (3), the hyperbolic dependence of viscosity on temperature is used, then instead of (20) or (21) we obtain the linear equation that was examined in [5].

Using the method of the qualitative theory of differential equations, we can show that if $S(\eta)$ in the problem for Eq. (20) is replaced by the minimum S_{min} or the maximum S_{max} value of this magnitude in the interval $0 \leqslant \eta \leqslant 1$, then the corresponding solutions θ_{min} and θ_{max} will represent the lower or upper boundaries, respectively, for the true profile of θ in the sense that for any η the following inequality is correct:

$$\theta_{min}(\eta) \leqslant \theta(\eta) \leqslant \theta_{max}(\eta). \tag{22}$$

(Physically this is perfectly obvious: increased dissipation in the flow with unchanged conditions on its boundaries leads to increased temperature at all points except boundary points.) The problems for θ_{min} and θ_{max} coincide with the one examined above, and in particular, there exist conditions when the second or both do not have a steady-state solution. Therefore the conclusion that a thermal burst may occur remains correct also when there is a longitudinal pressure gradient.

Furthermore, it can be shown that for $\varkappa > 0$ and for $\varkappa < -1$ the critical value β_* is a decreasing, and for $-1 < \varkappa < 0$ an increasing function of $|\varkappa|$. Along the lubricant layer of a real radial bearing the values of C and dp/dx change substantially, as is well known [10]. Therefore, when M or Ω increase, the conditions of occurrence of a thermal burst are fulfilled initially on limited sections along the circumference of the bearing, i.e., peculiar "thermal points" originate. Near these points the longitudinal temperature gradients abruptly increase, and in addition, in the general case the assumption that the temperature of the shaft and of the bearing housing is constant becomes unsuitable. In case the critical value is slightly exceeded, the heat released in sections of local thermal burst may be transferred not only transversely but also longitudinally, not leading to a substantial expansion of the mentioned sections and to the realization of a "global" thermal burst for the bearing as a whole. Therefore, under such conditions there have to exist steady-state regimes of a different nature entailing a substantial nonuniformity of the temperature distribution along the lubricant layer and in the materials of the bearing housing, possibly also of the shaft. This last may be the cause of considerable thermal strains and stresses and consequently of buckling and various kinds of distortions upsetting the balanced state of the entire system and the normal functioning of the bearing. Therefore, no further comment is needed concerning the applied importance of the investigation of the problem of thermal bursts in bearings.

NOTATION

a, thermal diffusivity; C, integration constant in (5) and (19); c, specific heat; h, thickness of the lubricant layer; I, regime parameters in (16) and (17); K, constant of the rheological law; k, parameter of Eq. (21); M, torque; m, exponent of the temperature dependence of viscosity; n, index of the rheological law; p, pressure; R, radius of the bearing; S, function in (20); T, temperature; U, ΩR; u, velocity; x, y, coordinates; α, heat transfer coefficient; β, parameter of problem (8); γ, characteristic value of the slip velocity; η, dimensionless transverse coordinate; θ, dimensionless temperature; $\varkappa$, parameter in (20); λ, thermal conductivity; μ, viscosity; ρ, density; ξ, independent variable in (21); σ, σ', parameters of the solution of problem (8); $\varphi = \exp\theta$; Ω, angular velocity of the shaft. Subscripts 0, 1, to the surfaces of the neck and of the shaft, respectively; *, critical values of parameters.

LITERATURE CITED

1. S. A. Kaganov, "Steady-state laminar flow of an incompressible liquid in a flat channel and in a cylindrical pipe with the friction heat and the temperature dependence of the viscosity taken into account," Zh. Prikl. Mekh. Tekh. Fiz., No. 3, 96-99 (1962).
2. E. A. Kearslay, "The viscous heating correction for viscosimeter flows," Trans. Soc. Rheol., 6, No. 2, 253-262 (1962).
3. S. A. Bostandzhiyan, A. G. Merzhanov, and S. I. Khudyaev, "Hydrodynamic thermal burst," Dokl. Akad. Nauk SSSR, 163, No. 1, 133-136 (1965).
4. S. A. Kaganov, "Flow of a liquid between rotating coaxial cylinders with the friction heat and the temperature dependence of the viscosity taken into account," Inzh.-Fiz. Zh., 8, No. 3, 307-310 (1965).
5. S. A. Bostandzhiyan and A. M. Stolin, "The critical conditions of the thermal regime of generalized Couette flow," Inzh.-Fiz. Zh., 17, No. 1, 86-94 (1969).
6. A. G. Merzhanov and A. M. Stolin, "Thermal theory of the flow of viscous liquids," Dokl. Akad. Nauk SSSR, 198, No. 6, 1291-1294 (1971).
7. A. G. Merzhanov, A. P. Posetsel'skii, A. M. Stolin, and A. S. Shteinberg, "Experimental realization of a hydrodynamic thermal burst," Dokl. Akad. Nauk SSSR, 210, No. 1, 52-54 (1974).
8. N. B. Aleksapol'skii and V. I. Naidenov, "Viscous burst upon nonisothermal motion of an incompressible liquid," Zh. Prikl. Mekh. Tekh. Fiz., No. 1, 94-97 (1980).
9. A. Cameron, Theory of Lubrication in Engineering [Russian translation], Mashgiz, Moscow (1962).
10. A. Sommerfeld, Mechanics of Deformable Bodies, Academic Press (1950).
11. O. Pinkus and S. S. Bupara, "Adiabatic solutions for finite journal bearings," Trans. ASME, J. Lubrication Technol., 101, No. 4, 111-116 (1979).
12. D. A. Frank-Kamenetskii, Diffusion and Heat Transfer in Chemical Kinetics, Plenum Publ. (1969).
13. S. Wada and H. Hayashi, "Hydrodynamic lubrication of journal bearings by pseudoplastic lubricants," Bull. JSME, 17, No. 8, 967-974 (1974).
14. G. Rajalingham, B. S. Prabhu, and B. V. A. Rao, "Steady state performance of a hydrodynamic journal bearing with a pseudoplastic lubricant," Trans. ASME, J. Lubrication Technol., 101, No. 4, 117-124 (1979).